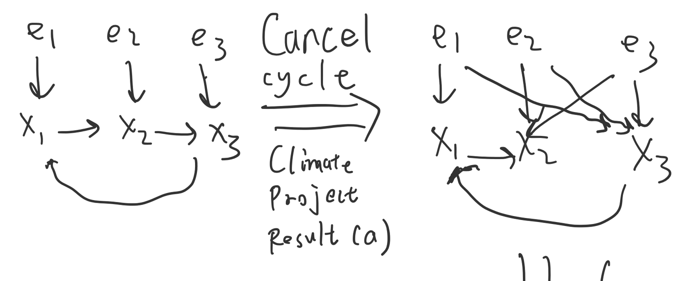
Sunday, October 27, 2024 8:36 PM

Stable Causal Graph is Complete Nonlinear ICA



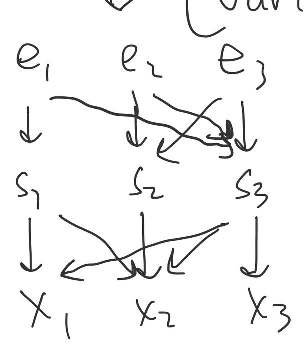
If not stable, e.g. $e_3 \to X_2$ explode

(self-loop can be handled by Lacerda et.al. 2012)

⇓ (variant of Patrik. et.al. 2023)

$J_X(e) = J_X(s) J_s(e)$ ⇐

$\frac{\partial x}{\partial e}$ $\frac{\partial x}{\partial s}$ $\frac{\partial s}{\partial e}$



← $s_3 = g_3(x_3)$, g_3 invertible

Cor. all of them are invertible, thus $J_X(e) = (I-B_1)^{-1} \cdot (I-B_2)^{-1}$, B_1, B_2 DAGs (Functional Equivalence)

Complete Nonlinear ICA is Stable Causal Graph

$X = g(e)$ since $J_g(e)$ is invertible and square, can always undergo LUP decomposition (and vice versa)

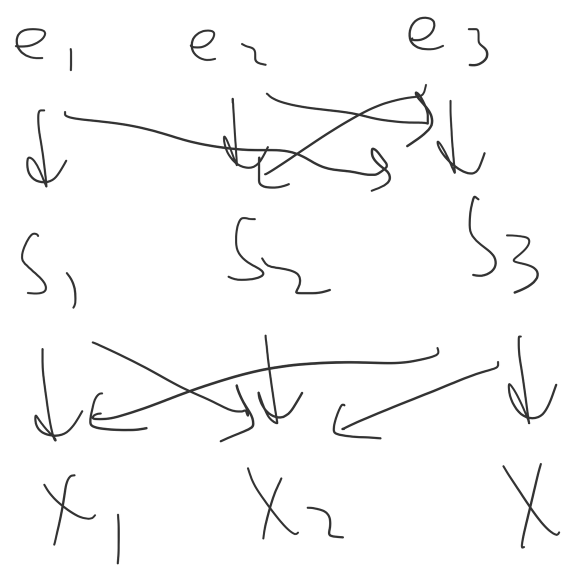
$J_g(e) = L\ U\ P$ (By definition of LUP decomposition)

L → lower triangle, U → Upper triangle, P → Permutation

L, U → both have non-zero diagonal

Note. If fix scaling and P, LUP decomposition is unique

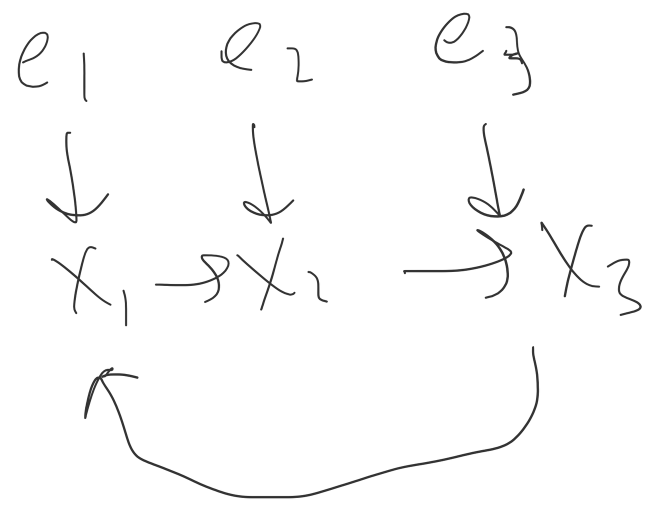
Thus L, U can express $(I-B)^{-1}$ (Functional Equivalence)



$J_s(e) = (I-U)^{-1}$ Up to a permutation P'

$J_X(s) = (I-L)^{-1}$

⇓ since $J_s(e), J_X(s)$ is bounded



← stable | Freely introduce self-loops by Lacerda et.al. 2012

TL;DR: Two Days "multiplication"

⟺ Stable Causal Graph

⟺ Complete Nonlinear ICA

$X_1 \to X_2 \to X_3$

$B: \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Markov Equivalence classes: $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Super matrix: $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$